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REVIEW

MARK GREAVES

The Philosophical Status of Diagrams

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The history of diagrams in logic and mathematics is a curious one. Through Euclid's *Elements*, diagrams occupied an important place in Western intellectual history for two millennia. Yet today formal characterisations of correct reasoning are almost entirely sentential. Why should this be so? Why should diagrams have been so completely superseded by sentences in geometry? And how did the present consensus arise that diagrams are simply out of place in logical or geometrical proofs?

This book addresses these questions, and in a clear and stimulating way. Geometrical and logical diagrams are discussed in separate self-contained sections, which are then brought together in a concluding chapter. The result is a rapid review of much of the history of each subject, but from a fresh and valuable perspective. In geometry, the book moves from Euclid, through Descartes' invention of analytic geometry, via Poncelet and projective geometry, to Pasch and the proto-formalism of Hilbert in the *Foundations*. In logic, the book discusses Euler diagrams against the background of the theory of the syllogism, before assessing the influence of Boole's algebraic approach to logic, Venn diagrams and Peirce's existential graphs.

Greaves's overall thesis is that diagrams fell into disuse as their representational limitations became evident, and that this process itself reflected changing views of the underlying subject matter. In geometry, he argues that the use of diagrams could accommodate the loss of its supposed early empirical grounding in surveying and cartography, and a move to the view that geometry is about idealised shapes. But the rise of analytic geometry suggested that geometrical claims could be represented, and represented with greater flexibility and generality, in equational form. And the traditional connection between the geometrical and the visual was finally and decisively broken by the development of non-Euclidean geometries, while the historically privileged status of Euclid's geometry was undermined by its location within an ordered hierarchy of geometries by Klein in the Erlangen Programme.

In logic, by contrast, a diagrammatic tradition was not so much undermined as inhibited from emerging at all. This, Greaves argues, can be largely traced to the influence, persisting into the late eighteenth century, of the Aristotelian syllogistic. For in Aristotle's treatment, the syllogism was a sentential representational form, reasoning with which relied on the user's recognition that the distribution of terms between the premises and the conclusion conformed to certain established schemata. Moreover, the subject-predicate form of the sentences was itself supposed to reflect the substance-attribute structure of the world. In the Aristotelian worldview, then, the sentential form was both a prerequisite for reasoning, and rationally appropriate to the underlying metaphysics.

Greaves plausibly suggests that two other related factors inhibited the emergence of diagrams in logic. The first was Aristotle's emphasis on the universality of logic. Its own generality notwithstanding, Euclid's geometry seemed to many to have a distinct subject matter, to which its diagrams gave visual access, even if the exact nature of that subject matter was itself a topic of debate. It may well have seemed opaque how there could be diagrams in logic at all if they had to be anything like Euclid's. The second factor was a broadly intensional view of concepts, made explicit during the early modern period, in which they were regarded as falling into a compositional hierarchy. If the concept MAN was partly composed of the concept ANIMAL, and that of ANIMAL was partly composed of the concept MORTAL BEING, then two questions naturally arose: how to diagram the claims, say, that 'All men are animals' and 'All animals are mortal' as in a syllogism in Barbara; and how to do so in a way that respected the nature of syllogistic inference.

These problems were partly solved by Euler in 1761, but not within the Aristotelian tradition. Rather, Euler construed concepts extensionally, and so read syllogisms as making claims as to the relations between classes of objects. This done, the way was open for him to represent the inclusion, exclusion or intersection of classes diagrammatically in terms of the inclusion, exclusion or intersection of circles. A syllogism in Barbara could thus be diagrammed as follows in Figure 1.

Euler's system was not without drawbacks, which included a risk of ambiguity in the interpretation of certain diagrams, difficulty in representing contradictory information, and difficulty in combining some diagrams (see Shin [1994], Ch. 2). But it served as the precursor and stimulus to more powerful systems in the nineteenth century: Venn's diagrams, Peirce's Existential Graphs, and Frege's Begriffsschrift. These systems reflected a desire to develop perspicuous notations to represent new developments in logic: specifically, in the case of Venn, Boole's new algebra, which rejected

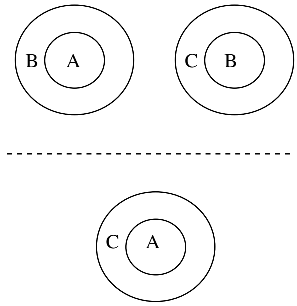

Figure 1. A subdivided population.

the subject-predicate analysis of propositions in favour of a much more inclusive characterisation of the relations between classes; in the case of Peirce, what we now think of as propositional logic (the alpha existential graphs) and first-order predicate logic (the beta graphs); and similarly for Frege. None of these systems caught on: they were all regarded as having various drawbacks—of representational adequacy, of logical power, or of ease of use. And for reasons already noted in the case of geometry, the late nineteenth century hardly offered a hospitable general environment for the use of diagrams to flourish.

This is an interesting approach to part of the histories of geometry and logic, and Grieves tells it well on the whole. One should bear in mind that, despite its title, *The Philosophical Status of Diagrams* is more a work of the (philosophically contextualised) history of ideas than a work of philosophy. Readers may be disappointed if they expect to find proper discussion of important philosophical questions relating to diagrams in formal contexts: for example, whether they can be of epistemic and not merely psychological value, and if so which types of diagram and how; whether they can properly be used in geometrical or logical proofs; and how we should understand the nature of diagrammatic representation.

The central thesis is that in general the history of diagrams in logic and geometry was heavily influenced by changing views as to the subject matter of those disciplines. This is plausible, even at first glance. That being so, the reader's interest naturally focuses on the analysis and critique of the specific systems themselves, taken as case studies. One wants to know: How exactly did this interaction between context, subject matter and representational form work itself out in each case? And should it have done so in this way? As it is, at just over 200 pages, the analysis of cases, such as those of non-Euclidean geometry and of Boole, inevitably stops short of philosophical bedrock, or substantive engagement with the views of others.

Given the importance of the specific analyses as case studies, it is disconcerting to find them clearly questionable in places. Let me give two small but indicative examples. First, Greaves claims that for Kant 'Concepts define the basic parameters under which our pure geometrical intuition can operate' (p. 58). But Kant's view is surely that our faculty of intuition is an independent source of representations quite distinct from the faculty of understanding (the source of concepts), and that neither faculty defines or constrains the other. To the extent that the faculty of intuition is 'defined' at all, it is in so far as our intuitions are ordered, non-conceptually, by Kant's so-called 'forms of intuition', space and time. Secondly, Greaves claims that 'Frege was not a geometer, and was not particularly interested in geometry except as a source of examples' (p. 63). But it surely demands mention that Frege wrote his doctoral thesis at Göttingen on geometry, conducted a vigorous debate with Hilbert in 1895–1903 on the foundations of geometry, and attempted c. 1924, at the very end of his life, to found the axioms of arithmetic on a geometrical basis given the failure of the *Grundgesetze*. It is plausible, in fact, that much of Frege's thought was structured throughout his life by a background dialectic between his developing views of arithmetic and geometry. In these and other cases, the desire to compress has rather militated against the book's clarity and balance.

Greaves rightly recognises the need to re-present Peirce's extremely subtle and interesting views on logic and mathematics to a general audience. But here again the discussion is too brief. Peirce's alpha existential graphs ('EG') are, as Greaves notes, a remarkable achievement: they are the first recognisably diagrammatic, sound and complete logical system—in fact equivalent to propositional logic. Moreover, they have some very interesting properties, notably that they are simultaneously readable in multiple ways (see Roberts [1973] and Shin [2002]). This multiple readability confers on them an immediacy and transparency of inference not possessed by their 2- and 5-functor sentential equivalents. Moreover, they have a direct sentential translation ('linear EG') which preserves all the rules of inference unchanged, and so enables direct comparisons to be made between the diagrammatic and sentential forms. And it is not uncommon to find that those familiar with both Peirce's graphs and sentential languages, this writer included, in fact prefer to teach propositional logic using the alpha graphs. Given these facts, one might expect some or all of the following in this book: an explanation of the semantics, syntax and rules of inference of the alpha graphs; examples of proofs; an exploration of the graphs' distinctively diagrammatic representational properties; a comparison with a sentential language; and a discussion of Peirce's crucial notions of 'icon' and 'symbol' in relation to the graphs. Unfortunately, the book barely touches on these topics, instead offering a brief and sometimes unclear descriptive survey.

To sum up: I have some important philosophical reservations about this book. But these apart, it is, overall, a valuable survey of an important but rather neglected topic.

References

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